

General Certificate of Education
January 2008
Advanced Level Examination



MATHEMATICS
Unit Further Pure 4

MFP4

Wednesday 30 January 2008 9.00 am to 10.30 am

For this paper you must have:

- a 12-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 Give a full geometrical description of the transformation represented by each of the following matrices:

(a) $\begin{bmatrix} 0.8 & 0 & -0.6 \\ 0 & 1 & 0 \\ 0.6 & 0 & 0.8 \end{bmatrix}$; (3 marks)

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. (2 marks)

- 2 It is given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + 4\mathbf{j} + 28\mathbf{k}$.

- (a) Determine:

(i) $\mathbf{a} \cdot \mathbf{b}$; (1 mark)

(ii) $\mathbf{a} \times \mathbf{b}$; (2 marks)

(iii) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. (2 marks)

- (b) Describe the geometrical relationship between the vectors:

(i) \mathbf{a} , \mathbf{b} and $\mathbf{a} \times \mathbf{b}$; (1 mark)

(ii) \mathbf{a} , \mathbf{b} and \mathbf{c} . (1 mark)

- 3 A shear S is represented by the matrix $\mathbf{A} = \begin{bmatrix} p & q \\ -q & r \end{bmatrix}$, where p , q and r are constants.

(a) By considering one of the geometrical properties of a shear, explain why $pr + q^2 = 1$. (2 marks)

- (b) Given that $p = 4$ and that the image of the point $(-1, 2)$ under S is $(2, -1)$, find:

(i) the value of q and the value of r ; (3 marks)

(ii) the equation of the line of invariant points of S . (3 marks)

4 The matrix \mathbf{T} has eigenvalues 2 and -2 , with corresponding eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ respectively.

(a) Given that $\mathbf{T} = \mathbf{U}\mathbf{D}\mathbf{U}^{-1}$, where \mathbf{D} is a diagonal matrix, write down suitable matrices \mathbf{U} , \mathbf{D} and \mathbf{U}^{-1} . (3 marks)

(b) Hence prove that, for all **even** positive integers n ,

$$\mathbf{T}^n = f(n) \mathbf{I}$$

where $f(n)$ is a function of n , and \mathbf{I} is the 2×2 identity matrix. (5 marks)

5 A system of equations is given by

$$\begin{aligned} x + 3y + 5z &= -2 \\ 3x - 4y + 2z &= 7 \\ ax + 11y + 13z &= b \end{aligned}$$

where a and b are constants.

(a) Find the unique solution of the system in the case when $a = 3$ and $b = 2$. (5 marks)

(b) (i) Determine the value of a for which the system does not have a unique solution. (3 marks)

(ii) For this value of a , find the value of b such that the system of equations is consistent. (4 marks)

Turn over for the next question

Turn over ►

6 (a) The line l has equation $\mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}$.

- (i) Write down a vector equation for l in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$. (1 mark)
- (ii) Write down cartesian equations for l . (2 marks)
- (iii) Find the direction cosines of l and explain, geometrically, what these represent. (3 marks)

(b) The plane Π has equation $\mathbf{r} = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$.

- (i) Find an equation for Π in the form $\mathbf{r} \cdot \mathbf{n} = d$. (4 marks)
- (ii) State the geometrical significance of the value of d in this case. (1 mark)
- (c) Determine, to the nearest 0.1° , the angle between l and Π . (4 marks)

7 The non-singular matrix $\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$.

- (a) (i) Show that

$$\mathbf{M}^2 + 2\mathbf{I} = k\mathbf{M}$$

for some integer k to be determined. (3 marks)

- (ii) By multiplying the equation in part (a)(i) by \mathbf{M}^{-1} , show that

$$\mathbf{M}^{-1} = a\mathbf{M} + b\mathbf{I}$$

for constants a and b to be found. (3 marks)

- (b) (i) Determine the characteristic equation of \mathbf{M} and show that \mathbf{M} has a repeated eigenvalue, 1, and another eigenvalue, 2. (6 marks)
- (ii) Give a full set of eigenvectors for each of these eigenvalues. (5 marks)
- (iii) State the geometrical significance of each set of eigenvectors in relation to the transformation with matrix \mathbf{M} . (3 marks)

END OF QUESTIONS